

Stable and metastable vacua in brane constructions of SQCD

Amit Giveon

*Racah Institute of Physics, The Hebrew University,
Jerusalem 91904, Israel
E-mail: giveon@phys.huji.ac.il*

David Kutasov

*EFI and Department of Physics, University of Chicago,
5640 S. Ellis Av., Chicago, IL 60637, U.S.A.
E-mail: dkutasov@uchicago.edu*

ABSTRACT: In a recent paper [1] we showed that $N = 1$ supersymmetric QCD in the presence of certain superpotential deformations has a rich landscape of supersymmetric and non-supersymmetric vacua. In this paper we embed this theory in string theory as a low energy theory of intersecting NS and D-branes. We find that in the region of parameter space of brane configurations that can be reliably studied using classical string theory, the vacuum structure is qualitatively similar to that in the field theory regime. Effects that in field theory come from one loop corrections arise in string theory as classical gravitational effects. The brane construction provides a useful guide to the structure of stable and metastable gauge theory vacua.

KEYWORDS: Intersecting branes models, Brane Dynamics in Gauge Theories.

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1. Introduction

Systems of intersecting Neveu-Schwarz (NS) fivebranes and D-branes provide a natural embedding of field theories into string theory. Many non-trivial aspects of the vacuum structure of various field theories appear naturally in these brane constructions. Conversely, field theoretic dynamics sheds light on the behavior of the branes in regions of parameter space where they are difficult to study using other means.

Most of the original work on this subject focused on theories with unbroken supersymmetry (see [2] for a review). More recently, interest turned to non-supersymmetric dynamics. In [3] it was found that $N = 1$ supersymmetric QCD (SQCD) has metastable vacua which can be reliably studied using field theoretic techniques in certain regions of coupling space. The brane realization of these vacua and many generalizations were discussed in [4–24]. As is familiar from other contexts, the region in the parameter space of brane configurations which can be reliably studied using classical string theory is different from the one in which the gauge theory analysis of [3] is valid. Nevertheless, it was found in [4–6, 15] that the pattern of metastable vacua in the brane systems is very similar to the gauge theory one, although the detailed dynamics is different.

In particular, in the field theoretic analysis of [3] one loop corrections to the potential for the light fields play an important role in giving mass to certain fields which classically have an exactly flat potential. It was pointed out in [15] that in the regime of validity of classical string theory this mass is due to gravitational attraction of the D-branes to the $NS5$ -branes. This phenomenon is reminiscent of worldsheet duality, whereby the interaction between D-branes can be studied by calculating the one loop partition sum of light

open strings when the D-branes are close, and by taking into account the exchange of light closed strings when they are far apart. As there, the two regimes are qualitatively similar.

In this paper we continue our investigation of supersymmetry breaking in intersecting brane systems, by generalizing the analysis of [15] to a configuration with a richer vacuum structure. On the field theory side this system is just SQCD with a particular superpotential perturbation turned on. Its vacuum structure in gauge theory is discussed in [1]. On the string theory side, the system we will study is that of [4–6, 15], with some of the branes rotated.

It was shown in [1] that in the field theory regime this system has a rich landscape of supersymmetric and (metastable) non-supersymmetric vacua. Our main purpose here will be to demonstrate that this landscape appears in the brane construction as well. As we will see, the brane picture provides a good way of identifying and studying both supersymmetric and non-supersymmetric ground states, albeit in a different regime in parameter space.

The plan of the paper is as follows. In section 2 we describe the brane configurations and study their ground states. In the gauge theory analysis of [1], classically the only ground states are supersymmetric. In the brane picture this is reflected in the fact that if we neglect the gravitational potential of the *NS5*-branes, all vacua of the brane system are supersymmetric.

In section 3 we include the gravitational potential of the *NS5*-branes and find a rich pattern of non-supersymmetric metastable states, in addition to the supersymmetric vacua of section 2. This pattern is very similar to that seen in gauge theory in [1]. In section 4 we comment on our results.

2. Brane configurations and supersymmetric vacua

To construct the brane configurations that we will study, it is convenient to decompose the $9 + 1$ dimensional spacetime as follows:

$$\mathbb{R}^{9,1} = \mathbb{R}^{3,1} \times \mathbb{C}_v \times \mathbb{C}_w \times \mathbb{R}_y \times \mathbb{R}_{x^7} . \tag{2.1}$$

The $\mathbb{R}^{3,1}$ labeled by (x^0, x^1, x^2, x^3) is common to all the branes, and is the arena for the dynamics of interest. The two complex planes \mathbb{C}_v , \mathbb{C}_w and real line \mathbb{R}_y correspond to

$$v = x^4 + ix^5, \quad w = x^8 + ix^9, \quad y = x^6 . \tag{2.2}$$

We will place in this spacetime extended branes that intersect on $\mathbb{R}^{3,1}$, as well as *D4*-branes localized at the intersection. The extended branes are *NS5*-branes and *D6*-branes filling $\mathbb{R}^{3,1}$ and the complex plane labeled by

$$w_\theta = v \sin \theta + w \cos \theta \tag{2.3}$$

in $\mathbb{C}_v \times \mathbb{C}_w$, and localized in the direction transverse to (2.3), $v_\theta = v \cos \theta - w \sin \theta$. We will refer to them as *NS $_\theta$* and *D6 $_\theta$* -branes, respectively. The *D6*-branes are further stretched in x^7 .

We will study brane configurations that contain two $NS5$ -branes and a stack of $D6$ -branes. In general, all three will have different orientations in $\mathbb{C}_v \times \mathbb{C}_w$ (i.e. different values of θ , (2.3)). By choosing the coordinates (v, w) appropriately, we can take one of the fivebranes to lie along the v axis. We will do that throughout the discussion, and refer to the corresponding $NS5$ -brane as an NS -brane (following customary notation [2]). In terms of the definitions above, this fivebrane corresponds to $NS_{\frac{\pi}{2}}$. The second fivebrane will be often taken to be NS_0 ; we will denote it by NS' , again following [2].

To summarize, the extended branes will be taken to stretch in $\mathbb{R}^{3,1}$ as well as the following directions:

$$\begin{aligned}
 NS &: & v, \\
 NS_{\theta'} &: & w_{\theta'}, \\
 D6_{\theta} &: & w_{\theta}, x^7.
 \end{aligned}
 \tag{2.4}$$

For general θ, θ' these branes preserve $N = 1$ supersymmetry in four dimensions. For some special values of the angles, the supersymmetry is enhanced to $N = 2$.

We will also consider $D4$ -branes which fill $\mathbb{R}^{3,1}$ and in the extra dimensions are stretched between pairs of the extended branes (2.4). Adding the $D4$ -branes leads to configurations which may or may not preserve supersymmetry. If the direction in which the fourbranes are stretched is y , (2.2), the full brane configuration is supersymmetric [2], but in general it is not. Our main purpose below will be to analyze the supersymmetric and non-supersymmetric vacua of brane configurations containing all the branes listed above, and compare the resulting vacuum structure to the $3 + 1$ dimensional effective field theory of the light modes localized at the intersection.

In the examples we will consider, that theory is SQCD with gauge group $U(N_c)$, N_f flavors of chiral superfields in the fundamental representation, Q_i, \tilde{Q}^i , and in general a non-zero superpotential $W_e(\tilde{Q}Q)$. We will also consider brane configurations whose low energy dynamics is described by the Seiberg dual magnetic gauge theory [25], with gauge group $U(N_f - N_c)$, N_f flavors of fundamentals q^i, \tilde{q}_i , gauge singlet chiral superfields M_j^i , which are magnetic duals of the electric meson fields $\tilde{Q}^i Q_j$, and superpotential

$$W_{\text{mag}} = \frac{1}{\Lambda} \tilde{q}_i M_j^i q^j + W_m(M),
 \tag{2.5}$$

where Λ is a scale familiar from studies of Seiberg duality. The magnetic quarks q, \tilde{q} will be taken below to be canonically normalized, while for M it will be convenient to use a different normalization.

The supersymmetric vacuum structure of brane configurations of the sort described above was analyzed and matched to field theory some time ago (see [2] for a review). Our main purpose here is to generalize these results to metastable supersymmetry breaking vacua. We will see that the brane construction is very useful in identifying and analyzing such states.

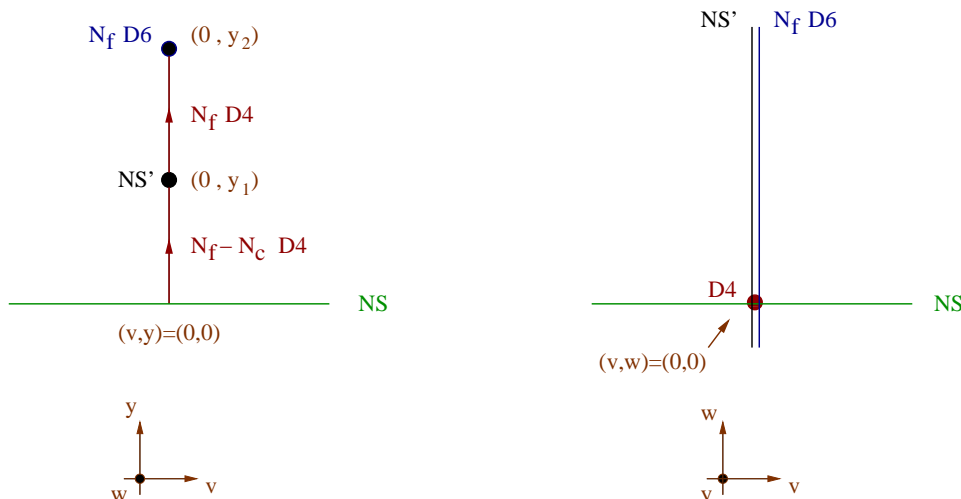


Figure 1: The magnetic brane configuration.

2.1 The magnetic SQCD brane configuration

We start with the brane configuration of figure 1, whose low energy limit is the magnetic gauge theory described above with $W_m(M) = 0$, [26, 27].

The figure on the left is a two dimensional slice of the brane configuration. The plane of the page is labeled by (one of the two components of) v and y (see (2.2)). The direction out of the page is w , and the slice is taken at the location of the NS -brane, $w = 0$. The figure on the right is a projection of the brane configuration on $\mathbb{C}_v \times \mathbb{C}_w$. In it, one can think of y as coming out of the page, and the different extended branes are located at different values of y . Comparing to the figure on the left we see that when viewed from above (in y), the order of the branes is: $D6$ -branes followed by the NS' -brane and then the NS -brane.¹

In addition to the extended branes discussed above, the configuration of figure 1 contains $D4$ -branes localized near the origin in the extra dimensions. The dynamics of these branes is the focus of our analysis. As reviewed in [2], the low energy theory on the $N_f - N_c$ $D4$ -branes stretched between the NS and NS' -branes is $N = 1$ SYM with gauge group $U(N_f - N_c)$. Strings stretched between these “color $D4$ -branes” and the N_f “flavor $D4$ -branes” which connect the $D6$ -branes to the NS' -brane give rise to N_f fundamental chiral superfields q^i, \tilde{q}_i . Finally, strings both of whose ends lie on the flavor $D4$ -branes give rise to gauge singlet superfields M_j^i . The magnetic quarks q, \tilde{q} and singlets M are coupled via the superpotential

$$W_{\text{mag}} = \frac{1}{\Lambda} \tilde{q}_i M_j^i q^j . \tag{2.6}$$

Many of the parameters of the brane configuration of figure 1 have an interpretation in the low energy effective field theory [2]. In particular, the classical $U(N_f - N_c)$ gauge coupling,

¹In this and subsequent figures we do not specify the value of θ for the $D6$ -branes, since it can be read off from the figures. In particular, in figure 1 the $D6$ -branes have $\theta = 0$ (see (2.3)).

g_{mag} , is given by

$$g_{\text{mag}}^2 = \frac{g_s l_s}{y_1}, \tag{2.7}$$

where l_s and g_s are the string length and ten dimensional string coupling, respectively, and y_1 is the distance between the $NS5$ -branes (see figure 1).

The magnetic superpotential (2.6) has flat directions corresponding to giving an arbitrary expectation value to the $N_f \times N_f$ matrix M_j^i while setting $q = \tilde{q} = 0$. This moduli space is realized geometrically in the brane construction via displacements of the $D4$ -branes stretched between the $D6$ -branes and the NS' -brane in figure 1 in the direction w , which is common to both types of branes.²

The precise relation between the displacement of the branes and the expectation value of M can be read off (2.6). A non-zero expectation value $\langle M_j^j \rangle$ gives rise via (2.6) to a mass $\langle M_j^j \rangle / \Lambda$ to q_j, \tilde{q}^j . Geometrically, this mass is due to the stretching of the fundamental string between the j 'th flavor brane which is displaced by the amount w_j , and the color branes in figure 1. Therefore, we conclude that the relation between the two is³

$$\frac{\langle M_j^j \rangle}{\Lambda} = \frac{w_j}{2\pi l_s^2}. \tag{2.8}$$

Another deformation of the magnetic gauge theory that will be of interest below is adding to (2.6) a linear superpotential,

$$W_{\text{mag}} = \frac{1}{\Lambda} \tilde{q}_i M_j^i q^j - m \text{Tr} M, \tag{2.9}$$

which is the magnetic dual of a mass term for the electric quarks Q, \tilde{Q} . In the brane picture this corresponds to displacing the $D6$ -branes relative to the NS' -brane in the v plane. We will normalize M such that displacing the $D6$ -branes to $v = v_2$ corresponds to adding to the superpotential the deformation (2.9) with [2]

$$m = -\frac{v_2}{2\pi l_s^2}. \tag{2.10}$$

The last deformation that we will consider corresponds to rotations of the $D6$ -branes in the (v, w) hyperplane. In figure 1 the $D6$ -branes are stretched in w , and one can ask what happens if we rotate them by an angle θ so that they are extended in w_θ , (2.3). This corresponds to adding to the superpotential (2.6) a mass term for M ,

$$W_{\text{mag}} = \frac{1}{\Lambda} \tilde{q}_i M_j^i q^j + \frac{\alpha}{2} \text{Tr} M^2, \tag{2.11}$$

with α related to θ as follows:

$$\alpha \Lambda = \tan \theta. \tag{2.12}$$

To prove this it is convenient to consider the brane configuration depicted in figure 2.

²In order to exhibit the full N_f^2 dimensional moduli space one needs to separate the $D6$ -branes in y ; see figure 29 in [2].

³Recall that the tension of the fundamental string is $T = 1/2\pi\alpha' = 1/2\pi l_s^2$.

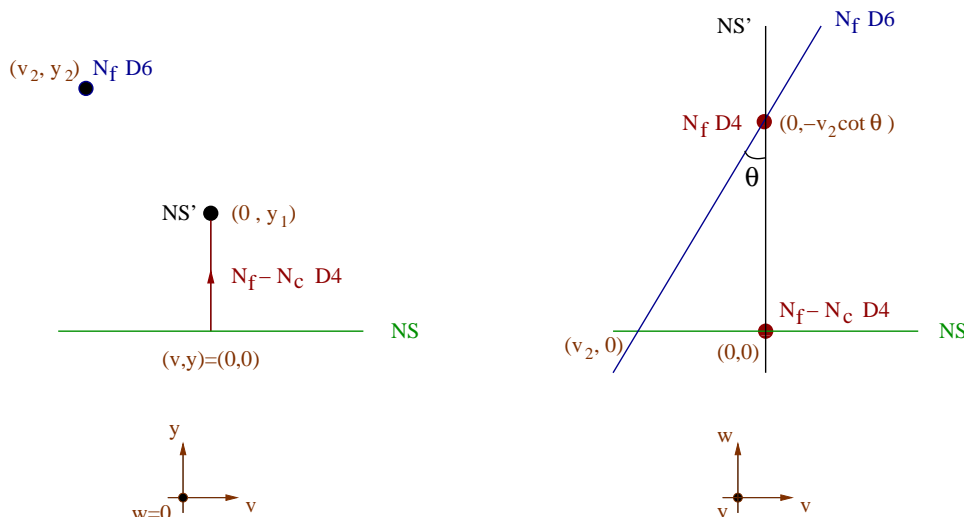


Figure 2: A deformed magnetic configuration.

Since this configuration can be obtained from that of figure 1 by a combination of a translation of the $D6$ -branes in v and a rotation, the corresponding superpotential has both of the deformations (2.9) and (2.11) turned on,

$$W_{\text{mag}} = \frac{1}{\Lambda} \tilde{q}_i M_j^i q^j + \text{Tr} \left(\frac{\alpha}{2} M^2 - m M \right) . \quad (2.13)$$

The supersymmetric vacuum depicted in figure 2 has $q = \tilde{q} = 0$, so that the gauge group $U(N_f - N_c)$ is unbroken, while the expectation value of M is determined by the F-term condition of (2.13):

$$M_i^j = \frac{m}{\alpha} \delta_i^j . \quad (2.14)$$

To calculate this expectation value in the brane picture we use (2.8) and the value of w for the flavor branes in figure 2:

$$M_i^j = -\frac{v_2 \cot \theta}{2\pi l_s^2} \Lambda \delta_i^j = m \Lambda \delta_i^j \cot \theta . \quad (2.15)$$

In the second equality we used the relation (2.10). Comparing (2.14) and (2.15) leads to (2.12).

The brane configuration of figure 1 has a number of interesting global symmetries. The $U(N_f)$ gauge symmetry on the N_f coincident $D6$ -branes descends in the low energy gauge theory to the diagonal $U(N_f)$ symmetry of magnetic SQCD. In this paper we will consider brane configurations that preserve it, as in [1]. It is easy to generalize the discussion to configurations that break the $U(N_f)$ symmetry, by separating the sixbranes. In particular, one can consider generalizations of the superpotential (2.13) in which m and α are more general matrices in flavor space.

The subgroup of the rotation group of the extra dimensions left unbroken by the brane configuration of figure 1 is $SO(2)_{45} \times SO(2)_{89} = U(1)_v \times U(1)_w$. These symmetries are

R-symmetries, and in comparing to field theory it is convenient to normalize the generators such that the supercharges have charge ± 1 . The R-charges of the various fields and parameters can then be deduced from the analysis of deformations above. The magnetic quarks have charge $(1, 0)$, M has charge $(0, 2)$, while the couplings m and α in (2.13) have charge $(2, 0)$ and $(2, -2)$, respectively. In particular, when both couplings are non-zero, the R-symmetry is completely broken.

Thus, we see that the brane configuration of figure 2 provides an example of a background which breaks R-symmetry, and it is of interest to construct metastable supersymmetry breaking states in it. In the rest of this section we will describe the supersymmetric vacua of the model. In section 3 we will turn to metastable states.

Since the brane configuration of figure 2 reduces in the infrared to the magnetic $U(N_f - N_c)$ gauge theory with the superpotential (2.13), one can compare its vacuum structure to that of the gauge theory. The gauge theory analysis was done⁴ in [1], where it was found that classical vacua are labeled by an integer

$$k = 0, 1, \dots, N_f - N_c . \tag{2.16}$$

For given k , the expectation values of M , q and \tilde{q} are given by

$$M = \begin{pmatrix} 0 & 0 \\ 0 & \frac{m}{\alpha} I_{N_f - k} \end{pmatrix} , \tag{2.17}$$

$$\tilde{q}q = \begin{pmatrix} m\Lambda I_k & 0 \\ 0 & 0 \end{pmatrix} . \tag{2.18}$$

In the k 'th vacuum the gauge symmetry is broken by the expectation value of q to $U(N_f - N_c - k)$. Thus, it is clear that the configuration of figure 2, in which the magnetic gauge group is unbroken, corresponds to $k = 0$.

The remaining vacua are easy to identify in the brane construction as well. One can take k of the N_f flavor $D4$ -branes and connect them to k of the $N_f - N_c$ color $D4$ -branes in figure 2, such that the resulting branes stretch directly from the $D6$ -branes to the NS -brane. To minimize their energy, these $D4$ -branes will move to $(v, w) = (v_2, 0)$, leading to the configuration of figure 3.

The $N_f - N_c - k$ $D4$ -branes stretched between the $NS5$ -branes in figure 3 give the unbroken $U(N_f - N_c - k)$ gauge group. The position in the w plane of the $N_f - k$ $D4$ -branes stretched between the $D6$ -branes and NS' -brane is related to the corresponding eigenvalues of M (2.17) via (2.8).

Thus, we see that the brane analysis reproduces the classical vacuum structure of magnetic SQCD with the superpotential (2.13). Quantum mechanically the $SU(N_f - N_c - k)$ gauge theory confines and breaks R-symmetry, such that there are $N_f - N_c - k$ distinct vacua with a mass gap. Furthermore, as shown in [1], for $N_f < 2N_c$ there are quantum supersymmetric vacua which are not seen classically, and are missed by the classical brane

⁴In [1] the gauge group was taken to be $SU(N_f - N_c)$; in the string embedding the baryon number symmetry is gauged as well.

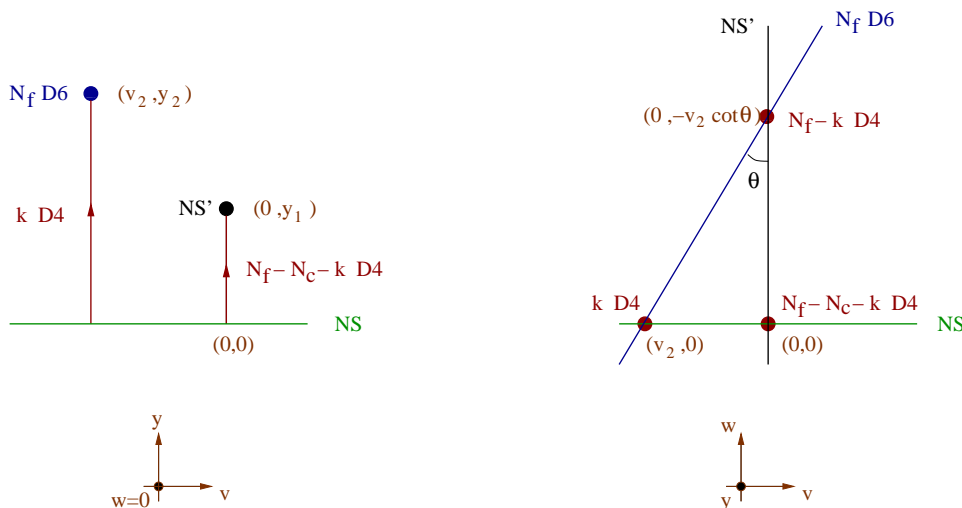


Figure 3: The classical supersymmetric vacua of the deformed magnetic configuration.

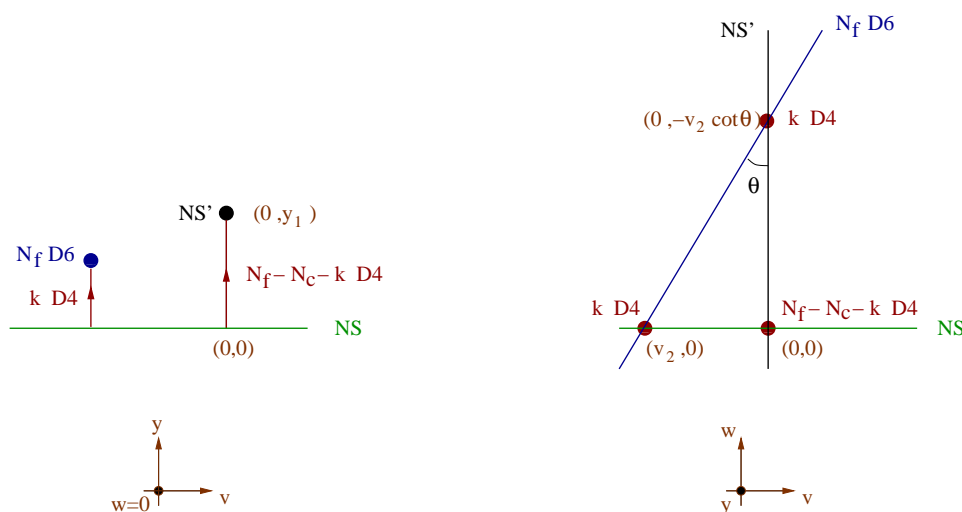


Figure 4: Another description of the supersymmetric vacua of figure 3.

construction. This is analogous to the fact that in the theory with $\alpha = 0$ considered in [3] there are no classical supersymmetric vacua at all, and the quantum vacua are not seen (at least naively) in the brane analysis [4–6].

2.2 Further deformations

The brane construction of the previous subsection can be deformed in a number of ways that do not qualitatively change the low energy physics. One involves the position of the $D6$ -branes in \mathbb{R}_y, y_2 , and in particular the process of taking the $D6$ -branes past the NS' -brane. Starting from the magnetic brane configuration of figure 3 and continuously changing y_2 to a value smaller than y_1 leads to the configuration depicted in figure 4.

In arriving at this figure we used the fact that when a $D6$ -brane crosses an $NS5$ -brane that is not parallel to it, a $D4$ -brane stretching between them is either created or destroyed

via a Hanany-Witten transition [28].

The low energy effective description of the brane configuration of figure 4 is similar to that of figure 3, with two differences. One is that it does not include the meson fields M . This is easiest to see by comparing the vacua with $k = 0$. In our previous discussion, the vacuum with $k = 0$ is described in figure 2. It includes N_f flavor $D4$ -branes stretched between the $D6$ and NS' -branes; the mesons M_i^j are the lowest lying modes of fundamental strings stretched between these branes. In the gauge theory description they are massive due to the non-zero value of α . In the brane picture their mass is due to the non-zero value of the angle θ .

On the other hand, in the configuration of figure 4, for $k = 0$ there are no $D4$ -branes stretched between the $D6$ and NS' -branes as a result of the Hanany-Witten transition mentioned above. Therefore, in this configuration, the meson fields M are absent. At energies much below the mass of M , one can think of the configuration of figure 4 as obtained from that of figure 3 by integrating this field out, and the behavior of the two systems is very similar. Above the mass of M they are different (see [1] for further discussion of this issue).

The second difference between the two brane configurations is of a more quantitative nature. The effective superpotential of the brane configuration of figure 4 is given by

$$W_{\text{mag}} = \text{Tr} \left[m_q \tilde{q} q - \frac{\alpha_q}{2} (\tilde{q} q)^2 \right], \quad (2.19)$$

which has the same qualitative form as what one would get by integrating out M in (2.13), but the coefficients m_q and α_q are different.

To determine m_q , consider the vacuum with $k = 0$ in figure 4. In this vacuum the expectation values of q , \tilde{q} vanish and the $U(N_f - N_c)$ gauge group is unbroken. As is clear from (2.19), the mass of q and \tilde{q} in this vacuum is given (up to an unimportant phase) by m_q . On the other hand, in figure 4 this mass corresponds to the energy of the lowest lying fundamental string stretched between the color $D4$ -branes and the N_f $D6$ -branes. That energy is given by $|v_2 \cos \theta|/2\pi l_s^2 = |m \cos \theta|$ (see (2.10)). Therefore, we conclude that

$$m_q = m \cos \theta. \quad (2.20)$$

Integrating out M from (2.13) gives instead

$$W_{\text{mag}} = -\frac{1}{\alpha\Lambda} \text{Tr} \left[\frac{1}{2\Lambda} (\tilde{q} q)^2 - m \tilde{q} q \right], \quad (2.21)$$

and hence

$$m_q = \frac{m}{\alpha\Lambda} = m \cot \theta \quad (2.22)$$

(see (2.12)). Thus, we see that under the Hanany-Witten transition, the mass m_q changes by a factor of $\sin \theta$. Determining α_q is more difficult, but it is easy to see that $\alpha_q \Lambda$ is a function of θ which vanishes at $\theta = 0, \pi/2$ and is symmetric under $\theta \rightarrow \frac{\pi}{2} - \theta$, which is not a property of the coefficient of $(\tilde{q} q)^2$ in (2.21).

Further displacing the $D6$ -branes past the NS -brane gives rise to a third brane configuration that describes the magnetic vacua above. This description can be obtained from that

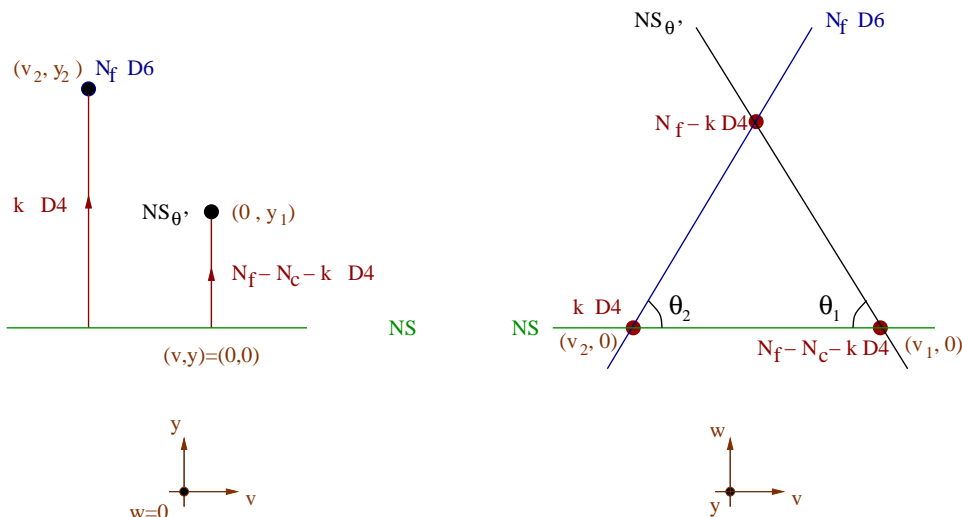


Figure 5: The magnetic configuration with rotated NS' -brane.

of subsection 2.1 by exchanging v and w and taking $\theta \rightarrow \frac{\pi}{2} - \theta$. In particular, the coupling α changes to $\alpha\Lambda = \cot \theta$ (compare to (2.12)). Thus, we see that Hanany-Witten transitions do not change the qualitative form of the low energy Lagrangian, but act non-trivially on the couplings.

Another deformation that does not qualitatively affect the low energy physics corresponds to a rotation of the NS' -brane by the angle θ' into the direction $w_{\theta'}$ (2.3). The resulting configuration is depicted in figure 5, where we introduced the notation $\theta_1 = \frac{\pi}{2} + \theta'$, $\theta_2 = \frac{\pi}{2} - \theta$.

The rotation of the fivebrane introduces into the dynamics an adjoint of $U(N_f - N_c)$ whose mass goes to infinity in the limit $\theta' \rightarrow 0$, but is finite for generic θ' [29]. This field couples to the magnetic quarks via a Yukawa superpotential. Integrating it out leads to a further contribution to the quartic superpotential for q and \tilde{q} , which goes to zero as $\theta' \rightarrow 0$ (i.e. $\theta_1 \rightarrow \frac{\pi}{2}$). For $\theta' = 0$ the superpotential is given by (2.21); for generic θ_1, θ_2 , the superpotential is given by (2.19), and the parameters m_q and α_q can be calculated as before. The former takes the form

$$m_q = \frac{(v_1 - v_2) \sin \theta_2}{2\pi l_s^2 \sin(\theta_1 + \theta_2)}. \quad (2.23)$$

As $\theta_1 \rightarrow \frac{\pi}{2}$ one recovers our previous result (2.22).

Note that m_q (2.23) diverges as $\theta_1 + \theta_2 \rightarrow \pi$. In this limit, the $NS_{\theta'}$ and $D6$ -branes become parallel which implies that the coefficient of M^2 in the superpotential goes to zero. The effective superpotential of M and q in this brane configuration has the form

$$W_{\text{mag}} = \frac{1}{\Lambda} \tilde{q} M q - m M + \beta (\tilde{q} q)^2. \quad (2.24)$$

The last term can be thought of as due to integrating out the adjoint of $U(N_f - N_c)$ discussed above. The coupling β depends on θ_1 (or equivalently on $\theta_2 = \pi - \theta_1$), and goes to zero as $\theta_1, \theta_2 \rightarrow \pi/2$. This is precisely the model that was studied recently in [30] in the

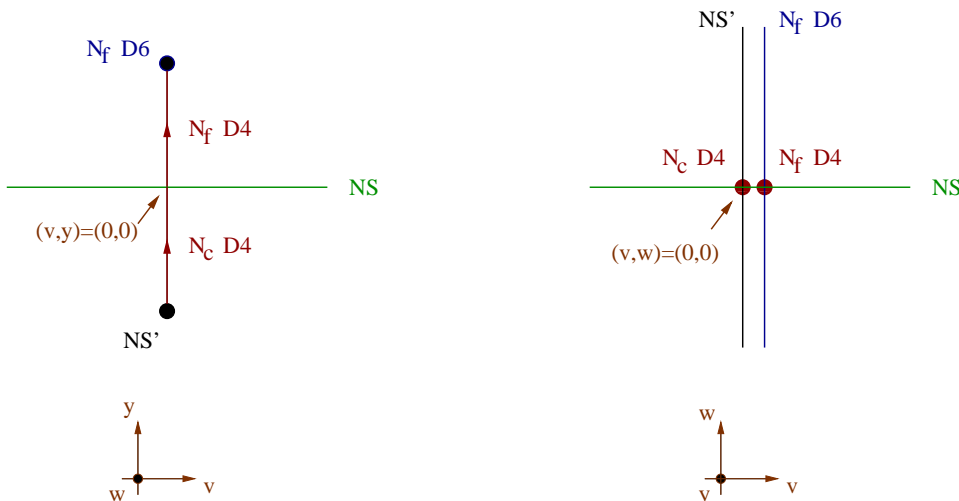


Figure 6: The electric configuration.

context of direct gauge mediation. Like the original model of [3], it does not have classical supersymmetric vacua, as is clear from the brane construction. Nevertheless, for generic $\theta_1 = \pi - \theta_2$ it breaks the R-symmetry. In the gauge theory this is due to the extra term in the superpotential (2.24), while in the brane construction it is clear from the geometry that $U(1)_v \times U(1)_w$ is broken in figure 5.

2.3 The electric configuration

The brane configurations discussed above give rise at low energies to magnetic SQCD, which is related by Seiberg duality [25] to an electric theory with gauge group $U(N_c)$. The corresponding brane configuration is presented in figure 6.

It involves N_c color $D4$ -branes stretched between the NS and NS' -branes, which give rise to $U(N_c)$ SYM theory, and N_f flavor $D4$ -branes stretched between the $D6$ and NS -branes, which give rise to N_f flavors of quarks in the fundamental representation of the gauge group, Q_i, \tilde{Q}^i . This construction, and in particular the relation between the configurations of figures 1 and 6, is discussed in [26, 27, 31, 32] and reviewed in [2].

Like in the magnetic description of the previous subsections, we can now translate the $D6$ -branes by an amount v_2 in the (45) plane and rotate them by an angle θ into the direction w_θ (2.3). The resulting brane configuration is described in figure 7, which is the electric analog of figure 3. Note that the two figures are identical, with the replacements $v \leftrightarrow w, \theta \leftrightarrow \frac{\pi}{2} - \theta, N_c \leftrightarrow N_f - N_c$.

In the electric gauge theory the deformation of figure 7 corresponds to turning on a superpotential for the quarks Q, \tilde{Q} ,

$$W_{\text{el}} = \text{Tr} \left[\frac{\alpha}{2} (\tilde{Q}Q)^2 - m\tilde{Q}Q \right] = \text{Tr} \left(\frac{\alpha}{2} M^2 - mM \right) . \quad (2.25)$$

The parameters m and α take the same values in terms of the geometric quantities as before, (2.10) and (2.12), respectively. As in the magnetic analysis, one can replace the

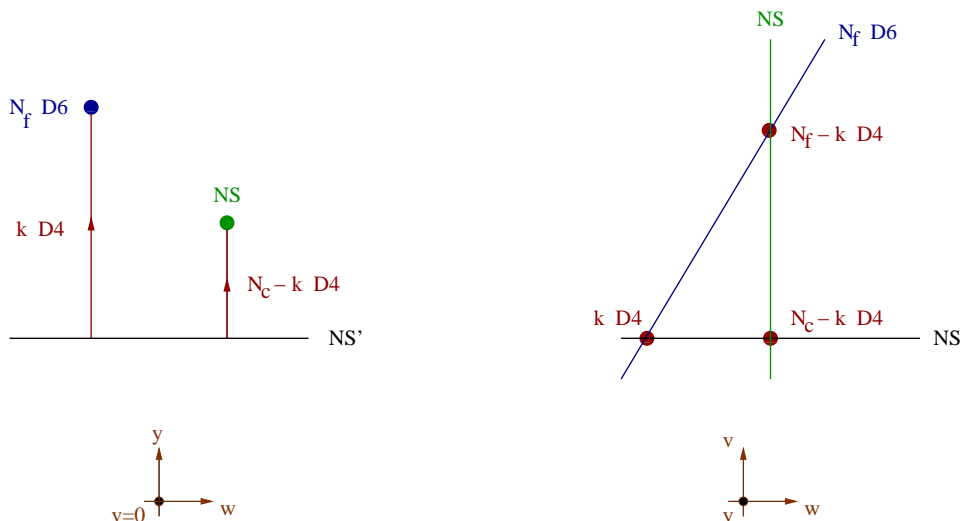


Figure 7: Supersymmetric vacua of the electric configuration.

superpotential (2.25) by [1]

$$W_{\text{el}} = -\frac{1}{\Lambda} \tilde{Q}^i N_i^j Q_j - \text{Tr} \left(\frac{\alpha_e}{2} N^2 - m_e N \right) . \quad (2.26)$$

The requirement that integrating out the massive fields N_i^j gives (2.25) implies that

$$\alpha \Lambda = \frac{1}{\alpha_e \Lambda} , \quad m = \frac{m_e}{\alpha_e \Lambda} . \quad (2.27)$$

The fields N_i^j are the lightest mode of open strings ending on the flavor branes, in complete analogy with the discussion of M_i^j in subsection 2.1. They become massless when $\alpha_e \rightarrow 0$, i.e. $\theta \rightarrow \frac{\pi}{2}$.

Classical supersymmetric vacua of the electric brane configuration are labeled by the parameter k indicated in figure 7, which runs over the range

$$k = 0, 1, \dots, N_c . \quad (2.28)$$

The unbroken gauge symmetry in the k 'th configuration is $U(N_c - k)$. All this is in agreement with the classical vacua found in gauge theory [1].

3. Metastable vacua

In the previous section we described certain supersymmetric intersecting brane configurations in type IIA string theory and compared them to the classical ground states of the low energy gauge theory on the branes. In [1] it was shown that when quantum effects are taken into account, additional non-supersymmetric metastable vacua appear in this gauge theory, at least when $N_f < \frac{3}{2} N_c$ and the coupling α is small. The existence of these vacua relies on a balance between the classical and one loop contributions to the effective potential of the light fields.

In the brane realization, it was shown in [15] (in a closely related setting) that in the regime where the dynamics of the branes is well described by classical string theory, the one loop field theory effects of [3] are replaced by the classical gravitational attraction of the $D4$ -branes to $NS5$ -branes. Thus, we expect the metastable vacua of [1] to appear in the systems described in section 2 when we take this attraction into account. The purpose of this section is to show that this is indeed the case.

It will turn out that the fivebrane whose gravitational potential plays a role in our problem is the NS -brane (2.4). In order to take its contribution into account we have to study the motion of the $D4$ -branes in the CHS geometry [33],

$$\begin{aligned} ds^2 &= dx_\mu dx^\mu + H(x^n) dx_m dx^m, \\ e^{2(\Phi-\Phi_0)} &= H(x^n), \\ H_{mnp} &= -\epsilon_{mnp}^q \partial_q \Phi. \end{aligned} \tag{3.1}$$

Here $\mu = 0, 1, 2, 3, 4, 5$; $m = 6, 7, 8, 9$; H_{mnp} is the field strength of the Neveu-Schwarz B field; $g_s = \exp \Phi_0$ is the string coupling far from the fivebranes. The harmonic function H is given by

$$H(r) = 1 + \frac{l_s^2}{r^2}, \tag{3.2}$$

with $r^2 = x_m x^m$. The background (3.1) is valid when $r \gg l_s$, and we will assume this throughout our discussion.

We will first consider a brane system in which the $D6$ and NS' -branes are separated in v and stretch in w , such as the configuration of figure 5 with $\theta_1 = \theta_2 = \frac{\pi}{2}$. This is the brane system studied in [4–6, 15]. In the flat space limit, $D4$ -branes stretched between the $D6$ and NS' -branes in this configuration can move in the w direction without any cost of energy. These are the brane analogs of the pseudo-moduli of [3].

In the NS -fivebrane geometry (3.1), the pseudo-moduli acquire a mass [15]. Indeed, if we hold the ends of the $D4$ -branes at a fixed value of w , their energy depends quadratically on this value (for small w). We next calculate this energy as a function of w for the case $y_1 = y_2$ in figure 5, and comment on the case $y_1 \neq y_2$. Then we tilt the branes by changing θ_1, θ_2 and find a locally stable equilibrium configuration. Finally, we use the above results to describe the pattern of locally stable magnetic brane configurations and match them to the gauge theory analysis of [1].

3.1 Parallel $D6$ and NS' -branes

In this subsection we consider the configuration of figure 5 with $\theta_1 = \theta_2 = \frac{\pi}{2}$. Thus, we have $D4$ -branes stretched between parallel $D6$ and NS' -branes and separated by the distance

$$\Delta x = |v_1 - v_2| \tag{3.3}$$

along the NS -brane. In [15] we calculated the energy density of such $D4$ -branes; here we would like to generalize the calculation to the case where the two ends of the fourbranes are displaced to $w \neq 0$.

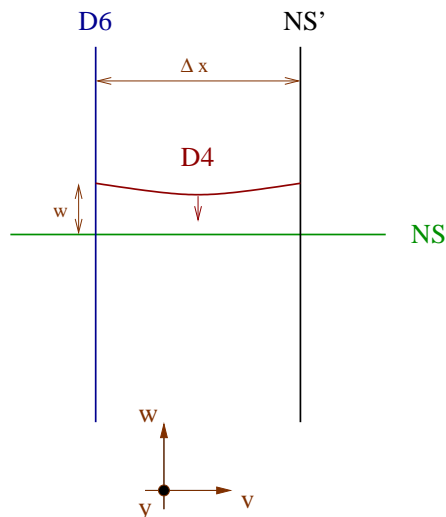


Figure 8: A $D4$ -brane displaced in w is attracted to the NS -brane.

To analyze this problem we consider the brane configuration in figure 8, in which a single $D4$ -brane is stretched between parallel $D6$ and NS' -branes, with its ends held fixed at an arbitrary value of w . We discuss in detail the case where the $D6$ and NS' -branes are at the same value of y , $y_1 = y_2 = y$, and then comment on the generalization to arbitrary y_j .

The energy of the $D4$ -brane in figure 8 can be read off the results of [15]. The only difference between the present situation and the one there is that the distance between the NS -brane and the other extended branes, y , should be replaced by

$$y_w = \sqrt{y^2 + |w|^2} . \tag{3.4}$$

Remembering that we set the number of NS -branes k to one, equations (3.11), (3.12) in [15] take the form

$$\Delta x = \frac{y_w^2}{l_s} \sin 2\theta_w + 2l_s \theta_w , \tag{3.5}$$

and

$$y_m = y_w \cos \theta_w , \tag{3.6}$$

where y_m is the smallest value of y along the $D4$ -brane.

The energy density of the $D4$ -brane is given by⁵

$$E(w) = 2\tau_4 \frac{y_m y_w}{l_s} \sqrt{H(y_m)} \sin \theta_w . \tag{3.7}$$

It is convenient to rescale E and define

$$\tilde{E} \equiv \frac{l_s}{2\tau_4} E , \tag{3.8}$$

⁵ τ_4 is the tension of a $D4$ -brane in flat spacetime.

which satisfies

$$\tilde{E}^2 = y_w^2 H(y_m) y_m^2 \sin^2 \theta_w = l_s^2 y_w^2 \sin^2 \theta_w + \frac{1}{4} y_w^4 \sin^2 2\theta_w . \quad (3.9)$$

Differentiating (3.9) with respect to w we find

$$\partial_w \tilde{E}^2 = l_s^2 \bar{w} \sin^2 \theta_w + l_s^2 y_w^2 (\sin 2\theta_w) \partial_w \theta_w + \frac{1}{2} y_w^2 \bar{w} \sin^2 2\theta_w + \frac{1}{2} y_w^4 (\sin 4\theta_w) \partial_w \theta_w . \quad (3.10)$$

In order to finish the calculation we need to calculate $\partial_w \theta_w$. This can be done by differentiating eq. (3.5) with respect to w , which leads to

$$\frac{(l_s^2 + y_w^2 \cos 2\theta_w) \partial_w \theta_w + 1}{2\bar{w} \sin 2\theta_w} = 0 . \quad (3.11)$$

Equation (3.11) implies that the sum of the last three terms in (3.10) vanishes, so that

$$\partial_w \tilde{E}^2 = l_s^2 \bar{w} \sin^2 \theta_w . \quad (3.12)$$

The only stationary point of (3.12) is $w = 0$. Indeed, if $w \neq 0$, (3.12) only vanishes for $\theta_w = 0, \pi/2$, and these values are unphysical (for generic Δx) according to (3.5), (3.6). This is reasonable, since if we move the $D4$ -brane in w , the attraction to the NS -branes provides a restoring force and we do not expect a stationary point at finite w .

Expanding the energy (3.7) around $w = 0$, one finds that the mode corresponding to displacement of the $D4$ -brane in w is massive, as expected,

$$E(w) = \frac{2\tau_4}{l_s} y \sqrt{l_s^2 + y_m^2} \sin \theta_0 + \tau_4 l_s \frac{\sin \theta_0}{y \sqrt{l_s^2 + y_m^2}} w \bar{w} + O(|w|^4) . \quad (3.13)$$

In equation (3.13), y_m is the smallest value of y along the $D4$ -brane when the latter is placed at $w = 0$, as in [15], and θ_0 is the value of θ_w at $w = 0$. Also, since this equation was obtained from a supergravity analysis, it is valid in the regime $y \gg l_s$, where it can be simplified:

$$E(w) = \tau_4 \Delta x + \frac{2\tau_4}{\Delta x} (\sin^2 \theta_0) |w|^2 + O(|w|^4) \simeq \tau_4 \Delta x + \frac{\tau_4 l_s^2 \Delta x}{2y^4} |w|^2 + O(|w|^4) . \quad (3.14)$$

To calculate the mass of the mode w we need to specify its kinetic term. This can be done in the flat spacetime approximation, to which the fivebrane geometry (3.1) only provides a small correction. In this approximation the $D4$ -brane is just a line segment of length Δx located at a fixed w . Thus, its kinetic term is given by the standard result

$$\mathcal{L}_k = -\frac{\tau_4}{2} \Delta x |\partial_\mu w|^2 . \quad (3.15)$$

Adding to this the potential energy density (3.14) we find that the mass of w is

$$m_w = \frac{l_s}{y^2} . \quad (3.16)$$

This mass is well below the string scale in the regime of validity of the supergravity approximation. Interestingly, it does not depend on the separation of the branes Δx (3.3). If this

behavior persisted to arbitrarily small Δx , this would be a problem in our analysis below of the vacuum structure of the configuration of figure 5, since as explained in [15], in that regime gauge theory should take over, and the leading contribution to the mass m_w should come from the one loop effect calculated in [3]. This effect leads to a mass $m_w^2 \sim \Delta x$ which goes to zero as $\Delta x \rightarrow 0$, and in particular is smaller than (3.16) for sufficiently small Δx .

The resolution of this is that in the brane construction of magnetic SQCD it is important to take $y_2 > y_1$ so that the configuration of figure 1 is non-singular. For $\Delta y = y_2 - y_1 \neq 0$, the previous discussion is generalized in two important ways. The kinetic term (3.15) involves the length of the $D4$ -brane, which is now $\sqrt{(\Delta x)^2 + (\Delta y)^2}$. Thus, in the limit $\Delta x \rightarrow 0$, the kinetic term approaches a finite limit,

$$\mathcal{L}_k = -\frac{\tau_4}{2} \Delta y |\partial_\mu w|^2 . \tag{3.17}$$

The second difference involves the potential energy (3.14). For $y_1 = y_2$ the coefficient of $|w|^2$ in the energy density E went to zero like Δx ; for $y_1 \neq y_2$ we expect it to vanish like $(\Delta x)^2$. The reason is that for $\Delta x = 0$ the configuration of figure 8 is supersymmetric for all w , and therefore the energy of the $D4$ -brane is independent of w . Furthermore, the limit $\Delta x \rightarrow 0$ is clearly non-singular and the dynamics is invariant under $\Delta x \rightarrow -\Delta x$, so the energy should be analytic in Δx near the origin.

Therefore, for $y_1 \neq y_2$ we expect the analog of the mass (3.16) to vanish like

$$m_w \sim \Delta x \tag{3.18}$$

as $\Delta x \rightarrow 0$. This contribution to the mass is smaller than the gauge theory one computed in [3], which goes like $\sqrt{\Delta x}$, as expected on general grounds.

3.2 Non-parallel $D6$ and NS' -branes

To analyze the system of figure 5 we need to generalize the discussion of the previous subsection to configurations where the $D6$ and NS' -branes are not parallel. To see what happens in these cases we turn to the configuration of figure 9, where the branes are placed as follows in $\mathbb{C}_v \times \mathbb{C}_w$:

$$\begin{aligned} D6 : & \quad v = -v_1 + tw , \\ NS' : & \quad v = v_1 - tw , \\ NS : & \quad w = 0 . \end{aligned} \tag{3.19}$$

Thus, they are tilted by equal and opposite angles. We will continue to assume that they are located at the same value of y . The generalization to non-equal angles and $y_1 \neq y_2$ is in principle straightforward.

The new element in figure 9 is the dependence of the distance between the $D6$ and NS' -branes along the NS -branes, Δx (3.3), on w :

$$\Delta x = 2|v_1 - tw| . \tag{3.20}$$

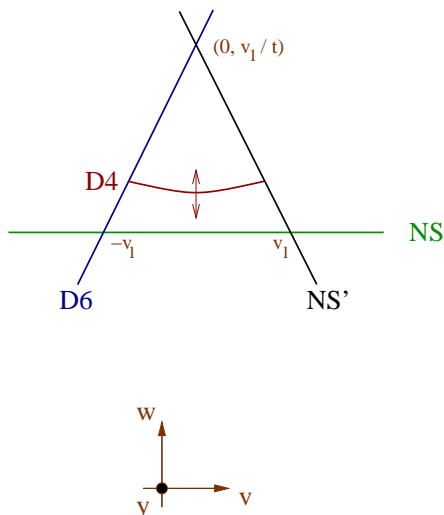


Figure 9: A $D4$ -brane stretched between tilted branes.

This leads to an extra contribution to $\partial_w \theta_w$ (3.11), which now takes the form

$$(l_s^2 + y_w^2 \cos 2\theta_w) \partial_w \theta_w + \frac{1}{2} \bar{w} \sin 2\theta_w - \frac{1}{2} l_s t \frac{\bar{t}\bar{w} - \bar{v}_1}{|tw - v_1|} = 0 . \quad (3.21)$$

Plugging this into (3.10) we find

$$\frac{\partial_w \tilde{E}^2 = l_s^2 \bar{w} \sin^2 \theta_w + 1}{2l_s t y_w^2 \sin 2\theta_w \frac{\bar{t}\bar{w} - \bar{v}_1}{|tw - v_1|}} . \quad (3.22)$$

One place where this must (and does) vanish is at $w = v_1/t$, where Δx (3.20), θ_w (3.5) and the energy density $E(w)$ (3.7) vanish. As is clear from figure 9, this is the global minimum of the energy, in which the $D4$ -brane approaches the intersection of the NS' and $D6$ -branes.

Physically one would expect to find another local minimum of the energy, at which the gravitational attraction of the $D4$ -brane to the NS -brane is precisely balanced by the force attracting the $D4$ -brane to the intersection of the $D6$ and NS' -branes. Without loss of generality we can take all the parameters v_1, w, t to be real and positive, and look for a solution with

$$v_1 > tw . \quad (3.23)$$

Setting the right hand side of (3.22) to zero leads to

$$\tan \theta_w = \frac{t y_w^2}{w l_s} . \quad (3.24)$$

To understand the form of the solution, let us look at the case where gravity is weak. To be more precise, we will consider the case where $v_1 \gg l_s$ and

$$v_1 l_s \ll y^2 \leq y_w^2 . \quad (3.25)$$

To understand this inequality, recall that in [15] it was shown that when $v_1 l_s$ reaches a value of order y^2 , a $D4$ -brane stretched between the NS' and $D6$ -branes becomes locally unstable. The inequality (3.25) is the requirement that we stay away from this regime of strong classical gravitational effects.

In this regime θ_w (3.5) is small, so (3.24) takes the form

$$\theta_w \simeq \frac{t y_w^2}{w l_s} . \tag{3.26}$$

Plugging this into (3.5) one finds the location of the local minimum of the effective potential,

$$w \simeq \frac{t y_w^4}{l_s^2 v_1} . \tag{3.27}$$

Plugging (3.27) back into (3.26) one can check that in the regime (3.25) θ_w is indeed very small.

Note that in deriving (3.27) we assumed that

$$t w \ll v_1 , \tag{3.28}$$

so we can approximate $\Delta x \simeq 2v_1$ in (3.20). In order for this to be the case, it must be that

$$t \ll \frac{v_1 l_s}{y_w^2} \ll 1 . \tag{3.29}$$

As is clear from figure 9, this implies that the angle between the $D6$ and NS' -branes is very small. It is not difficult to generalize the discussion to cases where that angle is not small.

One might also want to require that the local minimum occurs at a value of w much smaller than y . In that case one can replace y_w by y in (3.27). This leads to

$$w \simeq t \frac{y^4}{l_s^2 v_1} . \tag{3.30}$$

The requirement $w \ll y$ implies

$$t \ll \frac{v_1 l_s^2}{y^3} , \tag{3.31}$$

a more stringent bound than (3.29).

To summarize, we find that, as one would expect, the brane configuration of figure 9 has a local minimum where the ends of the $D4$ -brane are located at w given by (3.24). When the slope parameter t is small (3.31), the minimum is located at a small value of w (3.30).

3.3 Metastable states in the brane construction of SQCD

In the previous subsection we saw that a $D4$ -brane stretched between tilted NS' and $D6$ -branes, as in figure 9, has a locally stable configuration in which its ends are at a non-zero value of w given by (3.24), (3.30). In this subsection we would like to explore the implications of this for the brane realizations of SQCD discussed in section 2.

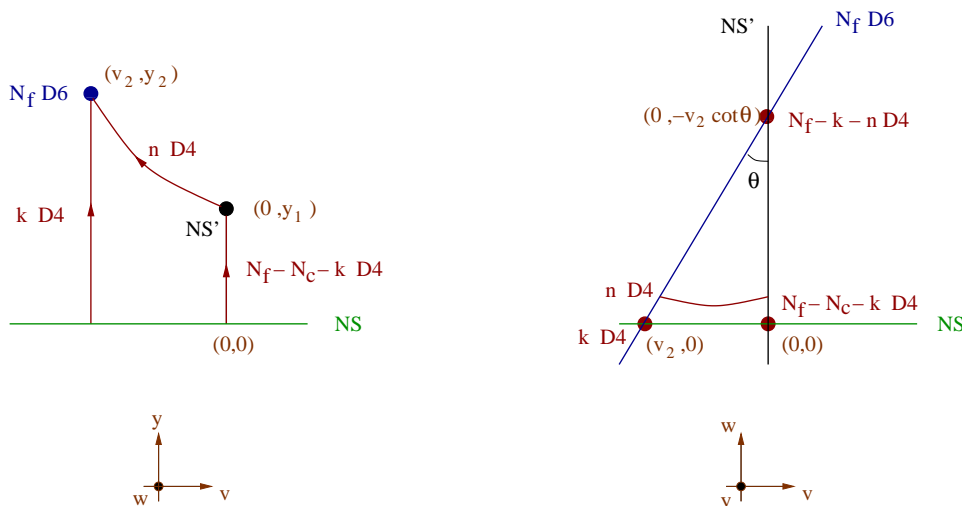


Figure 10: Metastable vacua of the deformed magnetic configuration.

For concreteness we will restrict to the magnetic brane configuration depicted in figure 3. In the k 'th supersymmetric vacuum there are $N_f - k$ $D4$ -branes stretched between the NS' and $D6$ -branes at $w = -v_2 \cot \theta$. We can move n of these $D4$ -branes towards the NS -brane to the local minimum of the potential found in the previous subsection. As we saw there, for small θ this minimum occurs at a small value of w (3.30). The resulting brane configuration is shown in figure 10.

The left figure shows the vicinity of $w = 0$ in the brane configuration. The endpoints of the n $D4$ -branes stretched between the NS' and $D6$ -branes are at a non-zero w , as can be seen in the right figure.

While the brane configuration of figure 10 is locally stable, it can decay to the supersymmetric ground states of section 2. For $N_f - N_c - k > 0$ there are two types of instabilities. One involves a process where the endpoints of the n flavor $D4$ -branes on the NS' -brane approach those of the $N_f - N_c - k$ color ones, the two types of branes connect and move to the intersection of the $D6$ and NS -branes. For $n \leq N_f - N_c - k$, the endpoint of this process is the supersymmetric vacuum of figure 3 with $k \rightarrow k + n$. Otherwise, some of the n flavor $D4$ -branes remain.

A second instability has the n $D4$ -branes moving to larger w , back to the configuration of figure 3. Of course, one can also consider processes where some of the $D4$ -branes approach the $NS' - D6$ intersection while the rest approach the $NS - D6$ one. For $N_f - N_c - k = 0$, the first type of instability is absent. Therefore, such vacua are more long-lived. More generally, this is the case for $N_f - N_c - k < n$.

In all the processes discussed above, the energy of the $D4$ -branes first increases and then decreases to the supersymmetric value. Thus, these are non-perturbative instabilities whose resolution involves tunneling. For fixed values of all the geometric parameters in the limit $g_s \rightarrow 0$, the lifetime of the metastable states goes like $\exp(C/g_s)$. The constant C depends on the particular state and would be interesting to calculate.

The metastable states of figure 10 are in one to one correspondence with those con-

structed in the magnetic gauge theory in section 3 of [1]. In that analysis, the effective potential of the magnetic gauge theory was found to have local minima in which the fields M and $\tilde{q}q$ have the form

$$M = \begin{pmatrix} 0 & 0 & 0 \\ 0 & M_n & 0 \\ 0 & 0 & \frac{m}{\alpha} I_{N_f - k - n} \end{pmatrix}, \quad (3.32)$$

and

$$\tilde{q}q = \begin{pmatrix} m\Lambda I_k & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad (3.33)$$

where the $n \times n$ matrix M_n is given (up to an order one factor) by

$$M_n \simeq \frac{\alpha\Lambda^3}{N_f - N_c} I_n. \quad (3.34)$$

These metastable states correspond to the ones in figure 10, with the parameters k and n that appear in both identified.

In fact, the gauge theory analysis of [1] is directly applicable to the brane configurations constructed here in a particular regime in their parameter space. In gauge theory it is natural to write the superpotential (2.13) as

$$W_{\text{mag}} = h\tilde{q}_i\Phi_j^i q^j - \text{Tr} \left(\frac{h\mu^2\Phi - 1}{2h^2\mu_\phi\Phi^2} \right) \quad (3.35)$$

where Φ is proportional to M and has a canonical kinetic term [1]. The couplings h , μ , μ_ϕ are given (up to order one factors) in terms of the brane parameters by (see (2.12) and [6])

$$h^2 = \frac{g_s l_s}{y_2 - y_1}, \quad \mu^2 = -\frac{v_2}{g_s l_s^3}, \quad \mu_\phi = \frac{\tan \theta}{g_s l_s}. \quad (3.36)$$

The analysis of [1] is valid for

$$\mu_\phi \ll \mu \ll m_s, \quad h \ll 1, \quad (3.37)$$

such that the physics is perturbative in h , μ_ϕ/μ , and takes place well below the string scale. Plugging in the values (3.36) leads to the constraints

$$\tan^2 \theta \ll \frac{v_2 g_s}{l_s} \ll g_s^2 \ll \left(\frac{y_2 - y_1}{l_s} \right)^2. \quad (3.38)$$

Thus, the field theory analysis is valid when θ and v_2/l_s are much smaller than g_s .

The classical brane construction generalizes the field theory discussion to the regime where the angle θ is of order one and the different length parameters in figure 3 are of order l_s or larger. In this regime the gauge theory analysis is not valid, but as we see the phase structure is essentially identical.

All the elements of the gauge theory discussion have direct analogs in the brane construction:

- (1) The n light flavors of $SU(N_f - N_c - k)$, denoted by $\varphi, \tilde{\varphi}$ in [1], correspond in the brane picture of figure 10 to fundamental strings stretched between the n flavor $D4$ -branes and the $N_f - N_c - k$ color ones.
- (2) The tachyonic instability found in gauge theory for $\mu_\phi < h\mu$ (see eq. (3.21) in [1]) is due in the brane construction to the fact that the ground state of the above fundamental strings is tachyonic when the angle θ and thus the distance between the endpoints of the color and flavor branes along the NS' -brane is sufficiently small.
- (3) The one loop effects that are necessary for stabilizing the metastable states in [1] are replaced in the brane picture by the classical gravitational attraction of the $D4$ -branes to the NS -brane, as in [15].

An interesting limit of the brane configuration of figure 10 is $\theta \rightarrow 0$ with the intersection point of the NS and $D6$ -branes $(v_2, 0)$ held fixed. The resulting configuration describes SQCD with the superpotential $W = -mM$ [4–6, 15]. It is natural to ask what happens to all the metastable states described above in this limit. The states with $1 \leq n \leq N_f - N_c - k$ become perturbatively unstable below a critical value of θ , as mentioned in point (2) above. Condensation of the tachyon mentioned there leads to a supersymmetric vacuum of the sort depicted in figure 3, with $k \rightarrow k + n$. For $n > N_f - N_c - k$, tachyon condensation leaves some flavor branes that cannot decay in this way. These appear to give rise to metastable states in the theory with $\theta = 0$.

One can exhibit all such vacua by taking $N_f - N_c - k = 0$, and letting n run over the range $n = 1, \dots, N_c$. For $n = N_c$ this procedure leads to the states considered in [3]. For $0 < n < N_c$ one finds additional states not considered in [3]. These states have the property that as $\theta \rightarrow 0$, $N_f - k - n$ $D4$ -branes in figure 10 go to infinity in w . One expects quantum corrections to modify the physics of these fourbranes, but since their dynamics takes place far from the n flavor $D4$ -branes that give rise to the metastable states, it is not clear that these effects should influence the metastable states. This is certainly the case in the gravity regime, where figure 10 is reliable, and it would be interesting to see whether such states exist in gauge theory as well.

4. Discussion

The fact that intersecting NS and D-brane constructions of the sort reviewed in [2] provide a useful guide for the analysis of supersymmetric ground states in various quantum field theories has been known for some time. The main conclusion of the present investigation is that this is the case for metastable non-supersymmetric ground states as well. We found that taking into account the gravitational attraction of the D-branes to the NS-fivebranes leads to a rich landscape of metastable states which are very similar to the corresponding gauge theory ground states.

This construction can be generalized in many ways discussed in the supersymmetric context in the past [2]. In particular,

- (a) Replacing the N_f $D6_\theta$ -branes by an NS_θ -brane corresponds in the low energy theory to gauging the $U(N_f)$ global symmetry.
- (b) Increasing the number of Neveu-Schwarz fivebranes leads to higher order polynomial superpotentials for the chiral superfields in the adjoint of $U(N_c) \times U(N_f)$. For instance, replacing the NS_θ -brane of point (a) by n_0 coincident NS_θ -branes and separating them in the transverse direction, leads to a superpotential of the form $W(M) = \sum_{n=1}^{n_0} \lambda_n \text{Tr} M^n$ for the adjoint of $U(N_f)$, M .
- (c) Replacing the NS' -brane in figure 1 (and subsequent figures) with a second stack of $D6$ -branes leads instead to an O'Raifeartaigh-type model with no gauge fields, of the type studied recently in [34].

These and other generalizations of the construction of this paper can be analyzed along the same lines, and presumably lead to a rich structure. It might be interesting to use such constructions to embed models of gauge mediation and their stringy generalizations in string theory.

Acknowledgments

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